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Strings in horizons, dissipation and a possible interpretation of the Hagedorn temperature

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Abstract. We consider the entanglement of closed bosonic strings intersecting the event horizon of a Rindler spacetime, and, by using some simplified (rather semiclassical) arguments and some elements of the string field theory, we show the existence of a critical temperature beyond which closed strings *cannot be in thermal equilibrium*. The order of magnitude of this critical value coincides with the Hagedorn temperature, which suggests an interpretation consistent with the fact of having a partition function that is ill defined for temperatures higher than it. Possible implications of the present approach for the microscopical structure of stretched horizons are also pointed out.

1 Introduction

A central issue in string theory at finite temperature is the meaning of the Hagedorn temperature [1], which may be related to a critical acceleration via the Unruh effect. Some authors argued that strings sufficiently near an event horizon should be accelerated so much that this critical value would be exceeded, and an infinite energy would be required for this purpose; consequently, the interpretation is that the string would slip into the black hole [2]. The main question we are addressing is how to describe this process.

Furthermore, on the other hand, a black hole interacts with systems that are at rest with respect to them (called fiducial systems) as a *dissipative* effective membrane placed on a neighborhood of the horizon. This idea was first used in astrophysics a long time ago and referred to as the “membrane paradigm” [3]. More recently, it was revisited in the context of string theory with the suggestive name of “stretched horizons” [4]; it has great interest for its actual physical meaning and microscopical picture. We are going to see here that these two issues as a result are found to be related in the context of strings.

In fact, stretched horizons should be placed at a distance from the event horizon, related to the l_s string length scale [5]. According to some references (e.g. [2, 6, 7]), the string is maximally accelerated at this distance, and the corresponding Unruh temperature is associated to the Hagedorn temperature. Here we propose that in this situation the event horizon intersects the worldsheet in order to show that, at the corresponding value of the critical temperature/acceleration, the string vacuum (seen by accelerated observers) is an entangled state of string modes living in the two causally disconnected regions. Notice that this *configuration* may be seen as two open strings ending precisely at the horizon and may be interpreted as a vertex diagram. Finally, we are going to argue that when a second quantized string theory is taken into account, the corresponding Hamiltonian to such a configuration (vertex) of the closed string intersected by the event horizon is actually related to dissipative processes when a string field description is adopted. Some results recently reported show that the infrared behavior of theories whose dual bulk-gravities contain a black brane is governed by hydrodynamics [8–10]. The most interesting result in this sense is the discovery of a universal value for the ratio of shear viscosity to entropy density [11]. So, although we are considering a Rindler spacetime, the fact that the configurations of a string at the horizon produce a “universal” dissipative Hamiltonian for an open string might reveal something about the microscopic nature of this universal hydrodynamic behavior from the string’s perspective.

The paper is organized as follows. In Sect. 2 we study the entanglement of the string vacuum and point out its relation with the microscopical structure of the stretched horizon. We use some semiclassical arguments to show the existence of a critical temperature at which these states arise. In Sect. 3 we use some elementary issues of light-cone string field theory to argue for the existence of a dissipative process beyond that critical temperature. In Sect. 4, we summarize and make some concluding remarks where briefly the possibility of identifying that critical value with the Hagedorn temperature (whose order of magnitude are

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in agreement) is discussed, and its eventual significance in order to explain the singular behavior of the partition function beyond this value.

2 Entanglement of a closed string in Rindler horizons and critical acceleration/temperature

Let us consider a bosonic closed string in light-cone gauge (LCG). The general solution for the transversal (physical) degrees of freedom is

$$X^i(t,\sigma) = x^i + 2\alpha' p^i t + \mathrm{i}\sqrt{\alpha'/2} \times \sum_{n\neq 0} \left(\alpha_n^i \mathrm{e}^{-2in(t-\sigma)} + \beta_n^i \mathrm{e}^{-2in(t+\sigma)}\right)/n\,, \tag{1}$$

where $\sigma \in [0, 2\pi] \sim S^1$, $t \in \mathbb{R}$, the worldsheet manifold is $W \sim S^1 \times \mathbb{R}_t$ and the worldsheet metric components corresponding to these coordinates are given by $g_0 = \mathrm{diag}(-1, 1)$. The field X defines an embedding of W into the spacetime M. In the presence of event horizons, the spacetime may be divided in two regions (for simplicity) denoted by $M^{\mathrm{in/out}}$, causally disconnected/connected with a certain class of observers.

The key idea in this simplified model is that, at least in a classical (and even semiclassical) sense, when the closed string (thought here as a classical extended object) intersects an event horizon at some point, no microscopical information goes through it, and this constitutes a critical regime. In this situation string vibrations are more and more red shifted so as to approximate the point of intersection with the horizon and are frozen at this point [12]. Let us now consider a convenient mathematical approach to this scenario.

For accelerated observers in the spacetime, the spacetime metric reads

$$\mathrm{d}S^2 = -R^2\,\mathrm{d}T^2 + \mathrm{d}R^2 + \mathrm{d}X_i\,\mathrm{d}X^i\,, \tag{2}$$

which is called the Rindler metric. On the other hand, the spatial coordinate R may be expressed as

$$R^2 = X^+X^- \tag{3}$$

in terms of the light-cone coordinates.

The Rindler spacetime is simply the wedge $X^+ > 0$, $X^- > 0$ of the Minkowski spacetime, and its boundary (horizon) is given by the surfaces $X^+ = 0$ and $X^- = 0$.[1] A Rindler observer, following a timelike curve in one of these four regions (say, the wedge I) in which the Minkowski spacetime is divided, only detects microscopic information coming from this one. In fact, the horizon surfaces correspond to infinite values of the proper time of these uniformly accelerated observers, $T(X^\pm = 0) = \pm\infty$. So despite that information may fall into wedge II (in the causal future of wedge I) crossing the null line $X^+ = 0$, it is not effectively observed by these Rindler observers in a finite proper time. In this precise sense one may refer to causal independence between the wedges.

The light-cone gauge fixing consists in taking $t = X^\pm$ as a time parameter of the string (LCG). In particular, if $t = X^+$ (notice that $t = X^-$ is an equivalent choice) we get

$$R^2 = tX^-\,, \tag{4}$$

which shows that the (closed) string intersects the horizon at least in $t = 0$ (so in worldsheet points such that $X^- = 0$). So, if the string worldsheet is $W \sim S^1 \times \mathbb{R}_t$, there is a complete circle where it intersects the horizon. This shows our initial assertion on the splitting in two regions $W^{\mathrm{in/out}}$. Thus, in agreement with [18, 19], the string lying behind the horizon $t < 0$ affects the string state by entanglement. So, by requiring continuity of the embedding $X : W \to M$, the string states in the horizon are given by the gluing condition,

$$\left(X^i_{\mathrm{in}} - X^i_{\mathrm{out}}\right)\big|_{t=0} |h\rangle = 0\,, \tag{5}$$

which actually constitutes a boundary state, since it is implemented at $t = 0$. By using the expansion in modes of (1) we may write

$$\beta_n^{\mathrm{in},i} - \mathrm{e}^{\mathrm{i}2n\lambda}\alpha_n^{\mathrm{out},i}|h\rangle = 0\,, \tag{6}$$

$$\alpha_n^{\mathrm{in},i} - \mathrm{e}^{-\mathrm{i}2n\lambda}\beta_n^{\mathrm{out},i}|h\rangle = 0\,, \tag{7}$$

$$x_0^{\mathrm{in},i} - x_0^{\mathrm{out},i}|h\rangle = 0\,, \tag{8}$$

where $x_0^{\mathrm{in},i}$ and $x_0^{\mathrm{out},i}$ denote both the center of mass coordinates. We have defined the respective string coordinates up to a shift, $\sigma_{\mathrm{out}} \equiv \sigma_{\mathrm{in}} + \lambda$, so λ is the relative twist between the respective ends of the gluing strings [13].[2] The solution of (6)–(8) may then be expressed as

$$|h\rangle = N_H \delta\left(x_0^{\mathrm{in},i} - x_0^{\mathrm{out},i}\right) \prod_{n>0,i} \mathrm{e}^{q_n \alpha_{-n}^{\mathrm{in},i}\beta_{-n}^{\mathrm{out},i}} \times \prod_{n>0,i} \mathrm{e}^{\bar{q}_n \beta_{-n}^{\mathrm{in},i}\alpha_{-n}^{\mathrm{out},i}} |0\rangle\,, \tag{9}$$

where $q_n = \mathrm{e}^{\mathrm{i}2n\lambda}$, and N_H is the proper normalizing constant.

Notice that in addition to the circle $t = 0$, the string intersects the horizon at the points $X^- = 0$. Classically, this condition defines a curve on the worldsheet $\sigma_H = S(t)$ for $t > 0$, which clearly separates the worldsheet in two parts, with topologies $[0, S(t)] \times \mathbb{R}_{t>0}$ and $[S(t), 2\pi] \times \mathbb{R}_{t>0}$ respectively, which are open sheets. So, in other words, if the world tube goes through the axis $X^- = 0$ along the time X^+, the worldsheet points are separated in two sets

[1] We refer to this as M^{out}.

[2] The signal of t is inverted in the horizon so, in order to define both fields $X^{\mathrm{in/out}}$ as parameterized by a positive time parameter, it is required that the right(in/out)/left(in/out) modes be defined such that they contribute both to the same term in the expansion of the general solution of (5).

$W^{\text{in/out}}$ defined such that the map $X^-(p \in W^{\text{in/out}})$ is positive/negative, respectively. In order to obtain the quantum states corresponding to this condition, once again, one might take the other possible choice consistent with the light-cone gauge (LCG), namely $X^- = t$, which clearly provides the same state (9).[3] The study of the quantum solutions of the condition $X^-|h\rangle = 0$ is an interesting issue in itself and will be investigated in another paper. However, here we are focused on showing that this situation may be described as genuine entanglement between string degrees of freedom, viewed as a two-dimensional field theory.

Let us study then the closed string entangled that we are talking about. From a strictly topological point of view, a closed string may be separated by the event horizon in two physical regions (simply connected or not), $W^{\text{in/out}}/X(W^{\text{in/out}}) = M^{\text{in/out}}$, which are clearly *open*.

Because this is a local free theory, $\mathcal{H}_{\text{closed}}$ can be trivially separated in a direct product of two independent Hilbert spaces $\mathcal{H}[S^{\text{in}}] \otimes \mathcal{H}[S^{\text{out}}]$, where $S^{\text{in}} \cup S^{\text{out}} = S^1$. To avoid (in principle) arbitrary confusion with standard open strings (defined by Dirichlet/Neumann combinations of boundary conditions) we name them "*c*-open strings". In fact, we can define the fields $X^{\text{in/out}}$ as the restriction of the field X to each wedge $W^{\text{in/out}}$ and quantize them with boundary matching conditions, $X^{\text{out}}|_{\partial S^{\text{out}}} = X^{in}|_{\partial S^{\text{in}}}$. The solutions simply are the restriction of the solution (1) to the respective intervals. So the vacuum state of a closed string may be written as a tensor product of two states in $\mathcal{H}^{\text{in}} \otimes \mathcal{H}^{\text{out}}$, namely $|0_{\text{in}}\rangle|0_{\text{out}}\rangle$. Then it is evident that in this situation the local degrees of freedom of the part of the string behind the horizon (W^{in}) produce *entanglement* and the fundamental state of the system is a mixed/entangled state, which clearly differs from the non-entangled vacuum state, where all the local degrees of freedom of the closed string are causally connected.[4]

So, in this sense we wish to show that the problem may be handled as in the Unruh–Hawking effect, where an ordinary field theory is quantized on a two-dimensional base manifold (the world-sheet) with an event horizon [15]. We are going to show indeed that when a closed string intersects the event horizon of accelerated observers, the string vacuum coincides with the boundary state (9), which entangles inner and outer string modes, and it is related to the inertial vacuum state through a Bogoliubov transformation.

To this purpose we may observe that the embedding $X : W \to M$, assumed to be smooth, induces a compatible worldsheet metric through the expression

$$g_{(W)ab} = \partial_a X^\mu \partial_b X^\nu g_{(M)\mu\nu} \,. \tag{10}$$

In particular, it is straightforward to verify that the worldsheet element of distance induced from (2) reads[5]

$$\mathrm{d}s^2 = -r^2\,\mathrm{d}\tau^2 + \mathrm{d}r^2 \,, \tag{11}$$

where the vector field $\tau^a := \nabla^a \tau (a = 0, 1)$ in the worldsheet is related to the d-vector tangent to the congruence of accelerated curves (Rindler observers), $\tau^\mu \equiv \nabla^\mu T$ ($\mu = 0, \ldots, d$), through the expression

$$\tau^\mu = \partial_a X^\mu \tau^a \,, \tag{12}$$

given by the pull-back corresponding to the string embedding. Similarly, we have defined the parameter $r^a := \nabla^a r$ as related to the Rindler spatial coordinate $R_\mu := \partial_\mu R$ through the pull-back

$$R^\mu = r^a \partial_a X^\mu \,. \tag{13}$$

This actually describes the worldsheet geometry in a neighborhood of the horizon.

Notice that the worldsheet horizon, defined by $r = 0$, actually describes the intersection of the string with a horizon in *the spacetime* for a time string parameter τ chosen in agreement with the proper time of a certain congruence of timelike curves in the spacetime associated with a particular class of (properly accelerated) observers. If $p \in H_W$, the time–time component of $g_{(W)}$ vanishes at this point, and then (since the pull-back is well defined, the character – timelike – of a vector is preserved by this) automatically the time–time component of $g_{(M)}$ also vanishes at $X(p)$,

$$\begin{aligned} 0 = g_{(W)\tau\tau}|_p &= g_{(W)ab}\tau^a\tau^b|_p \\ &= g_{(M)\mu\nu}|_{X(p)}\left(\partial_a X^\mu \tau^a|_p\right)\left(\partial_b X^\nu \tau^b|_p\right) = g_{(M)\tau\tau}|_{X(p)} \,. \end{aligned} \tag{14}$$

Then $X(p) \in H_M$, the horizon of the observers that follow the timelike curves generated by τ^μ. The same statement holds in the inverse sense, and therefore we see that the horizons of the respective congruences, related by (12), are such that $p \in H_W$ if and only if $X(p) \in H_M$. However, let us remark that the presence of H_W does not imply a coincident H_M, unless there do exist spacetime observers whose proper times are *synchronized* with the time string parameter (as is the case of the particular class of Rindler observers we are considering in this framework),[6] This is a useful property for the situation

[3] Notice that quantizing the condition $X^- = 0$, where X^- is thought of as the time parameter, corresponds to viewing the interval $[0, S(t)]$ as the evolution time rather than the (open) string range of the coordinate σ. This resembles the transformation that interchanges the closed into the open channel.

[4] We notice the remarkable fact that, in principle, an arbitrary separation of a closed string in two c-open strings may always be considered, but it is merely a formal construction except in the situation in which a closed string intersects a horizon. This will become clearer in the last section when we describe this as a vertex whose contribution shall be irrelevant except in the presence of the horizon.

[5] For more details, see the appendix.

[6] In [20, 21] there are examples where worldsheet horizons may not coincide with spacetime horizon crossings.

we are analyzing and considerably simplifies the analysis, since it allows us to deal with the problem of describing the string intersecting an event horizon directly in the *worldsheet*, rather than imposing this condition in the target space.[7] Thus, it is actually natural to interpret the relation (12) in a semiclassical sense (taking the expectation value in the right hand side), since the target vector τ^μ refers to the $d+1$-velocity of *classical* observers. One may therefore define classical frames (associated to observers in the spacetime) such that they detect the horizon $H_M/H_M \cap X(W) = X(H_W)$ in this way.

This picture is similar to the Unruh–Hawking scenario and we may follow the standard procedure, consisting in quantizing $D-2$ ordinary scalar fields on a two-dimensional (Lorentzian) manifold W in both accelerated and inertial frames and find the Bogoliubov transformation that relates the respective Fock spaces.

In the LCG, the physical degrees of freedom dynamics for a closed bosonic string are governed by the equation

$$\Box X^i = 0, \quad i = 1, \dots, d-1 \tag{15}$$

for the transversal coordinates of Rindler observers [12], which is also valid for inertial coordinates.

In particular, in order to quantize the system and to define the Hilbert space, we will take a worldsheet foliation in Cauchy surfaces (topologically S^1), whose parameter coincides with the proper time of a uniformly accelerated observer in the target space time.[8] In this case we use the Rindler coordinates (r,τ) locally induced from the Rindler ones as shown above, and the local string equation we have to solve is

$$\left(-r^{-2}\partial_\tau^2 + \partial_r^2 + r^{-1}\partial_r\right) X^i = 0 \tag{16}$$

in each patch (chart) of this type of coordinates in order to cover the worldsheet manifold. Solutions of this are well known [14]; then, by considering the proper conditions of smooth matching between the solutions on each chart and the periodicity conditions, we may construct a complete set of eigenfunctions $U_n(\tau,r)$ orthonormal with respect to the scalar product

$$\begin{aligned}(X_1,X_2) &= \sum_i \mathrm{i}\int_i X_1 \overset{\leftrightarrow}{\partial}_\tau X_2 \\ &= \sum_i \mathrm{i}\int_{r_i^-}^{r_i^+} \mathrm{d}r (X_1\partial_\tau X_2 - X_2 \partial_\tau X_1)\,, \end{aligned}\tag{17}$$

while the usual string solution in the Minkowski background is (1) (with the Minkowskian scalar product, $(X_1,X_2) = \mathrm{i}\int_{S^1} \mathrm{d}\sigma X_1 \overset{\leftrightarrow}{\partial}_t X_2$).

The general solution for the closed bosonic string in these coordinates may be expressed by

$$\begin{aligned} X^i(\tau,r) &= x^i + \mathrm{i}\sqrt{\alpha'/2} \\ &\times \sum_\eta \sum_{n\in Z} \left(b_n^{i(\eta)} U_n^{*(\eta)}(\tau,r) + \bar{b}_n^{i(\eta)} U_n^{(\eta)}(\tau,r)\right). \end{aligned}\tag{18}$$

In (18) the symbol $\eta = \text{in/out}$ (and $-\eta = \text{out/in}$) takes into account the fact that the worldsheet has a horizon, so it is divided into two causally disconnected regions or wedges. Following the standard procedure [14, 15], we can introduce the operators $d_n^\eta = \sum_{n\neq 0} P_n^\eta \alpha_n$, $\bar{d}_n^\eta = \sum_{n\neq 0} P_n^\eta \beta_n$, where P_n^η is a complete set of orthogonal functions, and now the operators $b_n^{(\eta)}$ and $d_n^{(\eta)}$ can be related by a Bogoliubov transformation:

$$\begin{aligned} b_n^{(\eta)} &= d_n^{(\eta)} \cosh(\epsilon_n) + \bar{d}_n^{(\eta)\dagger} \sinh(\epsilon_n) = G(\epsilon_n) d_n^{(\eta)} G^{-1}(\epsilon_n)\,, \\ \bar{b}_n^{(\eta)\dagger} &= d_n^{(\eta)} \sinh(\epsilon_n) + \bar{d}_n^{(-\eta)\dagger} \cosh(\epsilon_n) \\ &= G(\epsilon_n) \bar{d}_n^{(-\eta)\dagger} G^{-1}(\epsilon_n)\,, \end{aligned}\tag{19}$$

where the coefficients ϵ_n depend on the coordinate transformation parameters, and then they shall be related to the acceleration of the observer in the spacetime, and $G(\epsilon)$ reads

$$G(\epsilon) = \exp\left[\sum_\eta \sum_n \theta \epsilon_n \left(d_n^\eta \bar{d}_n^{-\eta} - d_n^{(\eta)\dagger} \bar{d}_n^{(-\eta)\dagger}\right)\right]. \tag{20}$$

It is clear that when the string does not intersect the horizon, the G transformation is trivial (it maps $|0_0\rangle_{\text{out}}$ into itself)[9]. In order to describe this, we have inserted the parameter θ, defined to be 1 if there is a horizon in the string worldsheet and 0 otherwise.

While the operators d_n^η and $\bar{d}_n^\eta$ annihilate the vacuum state $|0_0\rangle = |0_0\rangle_+ |0_0\rangle_-$, (referred to the two-dimensional Minkowski metric g_0), the operators $b_n^{(\eta)}$, $\bar{b}_n^{(\eta)}$ annihilate the vacuum:

$$|0(\epsilon)\rangle = G(\epsilon)|0_0\rangle\,, \tag{21}$$

which can be written as a $\mathrm{SU}(1,1)\times\mathrm{SU}(1,1)$ coherent state:

$$|0(\epsilon)\rangle = (1/Z)\exp\left[\theta \sum_\eta \sum_n (\tanh\epsilon_n)\left(d_n^{\dagger(\eta)} \bar{d}_n^{\dagger(-\eta)}\right)\right]|0_0\rangle\,, \tag{22}$$

where

$$Z = Z[\epsilon] \equiv \prod_n \cosh^2 \epsilon_n\,. \tag{23}$$

[7] In other words, the string coordinates are *operators*, which should be equal to the horizon coordinates, which are c-numbers. So this should be implemented on states rather than on operators, which should be similar to (5).

[8] In fact, the worldsheet manifold W may be decomposed as a collection of spatial one-dimensional manifolds $\Sigma_t \sim S^1$ and the scalar product is canonically defined by $(X_1,X_2) = \mathrm{i}\int_{\Sigma_t} X_1 \overset{\leftrightarrow}{\partial} X_2 = \mathrm{i}\int_{\Sigma_t} X_1 \partial_t X_2 - X_2 \partial_t X_1$, which allows one to quantize the field X^i and to construct the corresponding Hilbert/Fock space.

[9] In general entanglement theory $H = H^{\text{out}} - H^{\text{in}}$ and $[G,H] = 0$; then, if the worldsheet metric is horizon free, the Hamiltonian is $H \equiv \int_{S^1 \sim S^{\text{out}}} h\,\mathrm{d}r = H^{\text{out}}$, and we get $[G,H^{\text{out}}] = 0$.

We conclude this part by noticing that the boundary state (9) may be recovered from the expression of the fundamental state (22) for a suitable λ, as we should expect. In particular we get $\tanh \epsilon_n = q_n(\lambda)$, and the occupation number of the string modes is given by $N_n = \sinh^2 \epsilon_n$, which agrees with a Bose–Einstein distribution of string modes at the temperature $\mathrm{i}(4\lambda)^{-1}$. Therefore, the string twist produced by the horizon may be interpreted in terms of the temperature/acceleration of the Rindler observer.[10]

Clearly, this *vacuum state* consists of a condensate of string modes placed on the horizon region[11] which describes the critical point where the *transition* of closed (non-entangled) *to* c-open (entangled) string occurs. Then one may discuss the actual physical meaning of this transition in a semiclassical language.

In this particular situation, when the closed string (here thought of as a classical extended object) intersects the horizon at some point, the center of mass of the string is approximately at a distance $r_\mathrm{c} \equiv l_\mathrm{s}/2\pi$ from this intersection point, where the circumference length of the closed string is l_s.

On the other hand, the force/acceleration that has to be applied at the center of mass point in order to get a fiducial string[12] is given by the inverse of its distance [2] to the horizon, $a_\mathrm{c} \sim 1/r_\mathrm{c}$. Then, since the order of magnitude of the closed string circumference is $l_\mathrm{s} \sim \sqrt{2\alpha'}$, we finally conclude that $a_\mathrm{c} = 2\pi/l_\mathrm{s} \sim 2\pi/\sqrt{2\alpha'}$. Finally this acceleration may be related to a critical temperature via the Unruh effect, since the system at this point feels a thermal bath of temperature $T_\mathrm{c} \sim a_\mathrm{c}$. So, because the string is an extended object, when the acceleration of the center of mass point exceeds a_c, an event horizon intersects the worldsheet and the degrees of freedom behind it become causally independent, and the entanglement produced by these hidden degrees of freedom becomes non-trivial. This critical value is indeed similar to the Hagedorn temperature, $T_\mathrm{H} \sim 1/\sqrt{2\alpha'}$; so at least, it may be affirmed that *their orders of magnitude are in agreement.* This coincidence suggests a possible identification of both values and consequently a plausible model for the Hagedorn transition;[13], however this is merely a speculation that cannot be rigorously shown in the simplified context we are considering here.

Let us conclude this section by pointing out that this condensed state exists for accelerated (fiducial, for black hole horizons) observers, so that in the "membrane paradigm"[14] [3], or, in the modern language of strings and branes, we have a "stretched horizon" [4, 5]. So, here we wish to emphasize that this could constitute an appropriate microscopical *model* for this object,[15] which may be expressed as a boundary state (9) localized in the horizon.

In fact, it is believed that the stretched horizon is a hypersurface, that, for fiducial observers, behaves like an extended object with dissipative characteristics, a membrane. We are going to show in the next section how dissipative behavior naturally appears in the scenario presented here.

3 String field and dissipative behavior

If the coherent state (22) is assumed to be a microscopic description of the membrane paradigm, which has dissipative attributes [3], dissipative behavior should be expected to arise in this framework, and as a by-product of the arguments given in the previous section, it should be related to the Hagedorn scale.

The goal of this section is to argue for the dissipative behavior of strings in contact with a horizon discussed above when we consider a second quantized description, namely a string field approach. To this purpose we will only use very basic and generic properties of light-cone string field theory, which are supposed to be valid in this context [25], to argue for the existence of interaction terms between inaccessible/accessible (in/out) modes in a second quantized Hamiltonian. We will also use some simple arguments of "non-equilibrium thermofield dynamics" [22–24], a unified and canonical formalism that extends the thermofield dynamics (TFD) [17] to quantum dissipative systems. According to this, such a term would be evidence of dissipation, or more weakly: out of equilibrium processes, since the interaction between accessible/inaccessible modes describes energy-momentum exchange among them, which clearly causes loss of microscopic information into the inaccessible system.

In light-cone string field theory, the interaction diagrams are constructed by demanding the continuity of worldsheet embedding, which defines a set of gluing equations. The solution of the gluing equations are boundary states, which are vertex states living in the multi-string Hilbert space. For each vertex state there is a second quantized Hamiltonian that defines the string interaction. This is exactly what we have here. We have shown that Rindler observers see the closed string intersecting the horizon as an entanglement of strings, defined by (22) (or (9)). On the other hand, this state may be thought of as a squeezed *vertex* state $|V\rangle$ in the two-string Hilbert space $\mathcal{H}_\mathrm{in} \otimes \mathcal{H}_\mathrm{out}$, corresponding to two c-open strings, as discussed before. In fact, it is clear that the configuration described above

[10] In this case, it shall be thus interpreted as a purely imaginary twist $\lambda \equiv \mathrm{i}\beta/4$.

[11] A similar condensed state was found in [16], when a closed string approaches the null singularity of the *pp*-wave time dependent background.

[12] Since this situation is assumed to be stationary (i.e. the string does not fall into the causally prohibited region), the string has to be accelerated.

[13] this transition consists in the appearance of two (or more) causally disconnected regions in the worldsheet. This should finally depend only on the temperature rather than on a relative magnitude as the acceleration.

[14] For observers that remain stationary with respect to the black hole (and do not fall into it), the horizon region is effectively viewed as a membrane.

[15] For a discussion of the relation of this type of coherent states to macroscopical ones, see [17].

constitutes a *diagram* at tree level of two strings interacting, while their respective boundaries make contact on the horizon hypersurface and the gluing equations are defined in (5). This may be expressed as follows:

$$|V\rangle = \sum_{\mathbf{N}_1,\mathbf{N}_2} C(\mathbf{N}_1,\mathbf{N}_2)|\mathbf{N}_1,\mathrm{in}\rangle|\mathbf{N}_2,\mathrm{out}\rangle := |0(\epsilon)\rangle\,, \quad (24)$$

where the coefficients may be explicitly expressed by

$$C(\mathbf{N}_1,\mathbf{N}_2) = \langle \mathbf{N}_1,\mathrm{in}|\langle \mathbf{N}_2,\mathrm{out}|G|0_0\rangle\,, \quad (25)$$

where G is given by (22). Notice that in each term of (24) the quantum numbers $\mathbf{N}_1,\mathbf{N}_2$ coincide, since $C(\mathbf{N}_1,\mathbf{N}_2)=0,\ \forall \mathbf{N}_1 \neq \mathbf{N}_2$. By projecting these basis states in the position base, we get the wave functions for the $\mathbf{N}$th level:

$$\langle x|\mathbf{N}\rangle = \prod_n^{\infty} \Psi_{N_n}(x_n)\,, \quad (26)$$

where we are using the usual notation of light-cone string field theory, whose oscillation modes shall be built according to the expansion (18) and the component N_n of the vector $\mathbf{N}$ corresponds to the occupation number of oscillator n.

Let us define the functional field $\Phi^{\mathrm{in}}[p_1^+, x_1(\sigma), \sigma \in S^{\mathrm{in}}]$ of the string living in one side of the horizon and a functional $\Phi^{\mathrm{out}}[p_2^+, x_2(\sigma), \sigma \in S^{\mathrm{out}}]$ of the string living in the other side. The respective light-cone momenta are p_1^+ and p_2^+ and the expansion of the fields in position space is

$$\Phi^{\mathrm{in}}(p_1^+, x_1(\sigma)) = \frac{1}{\sqrt{|p_1^+|}} \sum_{\mathbf{N}} A_{\mathbf{N}}^{\mathrm{in}}(p_1^+) \prod_n^{\infty} \Psi_{N_n}(x_n)\,,$$
$$\Phi^{\mathrm{out}}(p_2^+, x_2(\sigma)) = \frac{1}{\sqrt{|p_2^+|}} \sum_{\mathbf{N}} A_{\mathbf{N}}^{\mathrm{out}}(p_2^+) \prod_n^{\infty} \Psi_{N_n}(x_n)\,, \quad (27)$$

where the second quantized operators $A_{\mathbf{N}_1}^{\mathrm{in}}(p_1^+)$, $A_{\mathbf{N}_2}^{\mathrm{out}}(p_2^+)$ are string creation operators for $p^+ < 0$ and string annihilation operators for $p^+ > 0$.

In a functional representation, the interaction Hamiltonian related to the configuration of the two c-open strings on the horizon (i.e. for Rindler observers) is bilinear in the fields $\Phi^{\mathrm{in}}(p_1^+, x_1(\sigma)$ and $\Phi^{\mathrm{out}}(p_2^+, x_2(\sigma))$. It may be written (up to a suitable coupling constant) in the number basis as

$$H_I = \int \mathrm{d}p_1^+\,\mathrm{d}p_2^+ \sum_{\mathbf{N}_1,\mathbf{N}_2} C(\mathbf{N}_1,\mathbf{N}_2) A_{\mathbf{N}_1}^{\mathrm{in}}(p_1^+) A_{\mathbf{N}_2}^{\mathrm{out}}(p_2^+)\,, \quad (28)$$

which describes the contact interaction between the two strings encoded in the (vertex) state (24), with the $C(\mathbf{N}_1,\mathbf{N}_2)$ coefficients given by (25).

Then this Hamiltonian typically describes a dissipative/out of equilibrium system; since for observers (that detect the horizon) in one of these sides, the system/field in the other (hidden) side may be considered "non-physical". Here both systems are physically realized, but as observed by Israel [18], the important fact is that they are causally disconnected. Thus, this structure, with a coupling between physical and non-physical modes, is analogous in form to the non-equilibrium-TFD, characterized by a Hamiltonian that includes a coupling between the physical and non-physical modes. It is remarkable that in the presence of horizons, while quantum fields of particles produces a scenario similar to equilibrium TFD [15, 18], in this approach, owing to the extended nature of strings, free string fields behave more according to non-equilibrium TFD.

Finally, for completeness, from (25) we may explicitly verify that for $\theta = 0$, corresponding to considering the string field in a region distant from the horizon, so that for inertial observers $C(\mathbf{N}_1,\mathbf{N}_2) = 0,\ \forall \mathbf{N}_1,\mathbf{N}_2 \neq 0$. Thus, in this case the contact Hamiltonian reads

$$\int \mathrm{d}p_1^+\,\mathrm{d}p_2^+ C(\mathbf{0}_1,\mathbf{0}_2) A_{\mathbf{0}}^{\mathrm{in}}(p_1^+) A_{\mathbf{0}}^{\mathrm{out}}(p_2^+)\,, \quad (29)$$

which is irrelevant for the closed string dynamics observed in inertial coordinates. In fact, this encodes a sort of product between Φ^{in} and Φ^{out} that precisely shall be equivalent to a linear term in the closed string field Φ, which clearly does not contribute to its own evolution equation.

4 Concluding remarks

We proposed a simplified model in which the transition closed $\to c$-open strings, occurring when the string intersects a horizon (and the center of mass acceleration is $a_{\mathrm{c}} \sim 2\pi(l_{\mathrm{s}})^{-1}$), may be interpreted in this approach as a limit for the equilibrium of the system beyond which there is a transition equilibrium/dissipation rather than a standard phase transition. This critical acceleration may be related to a temperature via the Unruh effect; so the cut temperature is $T_{\mathrm{c}} \sim 1/\sqrt{2\alpha'}$, which remarkably *coincides* with the order of magnitude of the Hagedorn temperature T_{H}. It then seems to be natural to hypothesize the possible identification of these critical temperatures and consequently interpret T_{H} as being a limit temperature for a system of *non-interacting strings* in equilibrium.

It is believed that the Hagedorn regime should be explained in the context of strong coupling with the background spacetime, since the Hagedorn singularity arises by summing over highly excited modes in the partition function. However, this singularity appears *even* for free strings in a Minkowski spacetime and in this approach we precisely get a simplified model in which a singularity is also present, but with some new ingredients related to entanglement and dissipation, which, rather than to be viewed as an explanation, may shed some light on physics in the Hagedorn phase. In fact, we assumed weakly coupled strings in nearly flat background spacetimes such that the back-reaction is controlled. These conditions are also valid for very massive black holes, where near the horizon gravity is weak and the metric may be approximated by a Rindler one [4, 5].

Let us remark that this hypothesis would explain why the partition function corresponding to an *equilibrium ensemble* is ill defined beyond this value (in particular, it diverges). In addition, these results provide a physical mechanism to enforce the approaches where the distance of the horizon $(a_c)^{-1}$ constitutes a cut-off in order to evaluate thermal observables (e.g free energy) and obtain finite results. In fact, closer degrees of freedom could not be considered in equilibrium and consequently these observables would be improperly defined in such regions.

Finally, we have also found an horizon filling boundary state, with many of the properties that one should wish to recover in a stretched horizon model.

Appendix: Induced worldsheet metric in a Rindler spacetime

The string embedding $X\colon W \to M$ is assumed to be smooth for simplicity. τ^μ $(\mu = 0, \ldots, d)$ is the d-vector tangent to the accelerated curve and its affine parameter is T, and we let $R_\mu \equiv \partial_\mu R$.

Let us introduce the vector fields $\tau^a := \nabla^a \tau$ and $r^a := \nabla^a r$ $(a = 1, 2)$ in $T_p W$ and define them through the pull-back

$$\tau^\mu = \tau^a \partial_a X^\mu \,, \tag{A.1}$$

$$R^\mu = r^a \partial_a X^\mu \,. \tag{A.2}$$

On the other hand, the worldsheet metric is consistently induced via this map from the metric (2)

$$g_{(W)ab} = \partial_a X^\mu \partial_b X^\nu g_{(M)\mu\nu} \,; \tag{A.3}$$

then the line element reads

$$\mathrm{d}s^2 = -R^2 \mathrm{d}\tau^2 + \mathrm{d}r^2 \,. \tag{A.4}$$

This may be verified by virtue of the relations

$$\tau^\mu \tau^\nu g_{(M)\mu\nu} = \tau^a \tau^b g_{(W)ab} = -R^2 \,, \tag{A.5}$$

$$R^\mu R^\nu g_{(M)\mu\nu} = r^a r^b g_{(W)ab} = 1 \,, \tag{A.6}$$

$$\tau^\mu R^\nu g_{(M)\mu\nu} = \tau^a r^b g_{(W)ab} = 0 \,. \tag{A.7}$$

Finally, by using (A.2) and the definition of the Rindler coordinates (2), we may notice that $\partial_\tau R = \tau^a \partial_a R = \tau^a (\partial_a X^\mu \partial_\mu R) = \tau^\mu \partial_\mu R = \frac{\partial R}{\partial T} = 0$. This implies that $R = R(r)$, and it may be chosen as $R(0) = 0$; thus, near $r = 0$ the worldsheet metric takes the form (11):

$$\mathrm{d}s^2 = -\kappa r^2 \mathrm{d}\tau^2 + \mathrm{d}r^2 \kappa = \text{const.} \,, \tag{A.8}$$

and κ is a positive constant, due to the signature, which may be taken to be one.

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